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Technical Notes

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Explicit Kutta Condition for an Unsteady Two-Dimensional Constant Potential Panel Method

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Introduction

EXPERIMENTALISTS differ in their conclusions for the boundaries of validity of a Kutta condition applied at the trailing edge of oscillating airfoils.[1-4] However, in numerical work it is recognized that a Kutta condition should be applied at least for oscillations of low reduced frequency and where the angle of attack of the airfoil is not sufficient to cause trailing-edge separation (see Ref. 4, p. 479).

Here, an explicit Kutta condition is described that was implemented in a time-domain constant potential panel method for two-dimensional airfoils in unsteady motion. The panel method was written to calculate thrust and propulsive efficiency from oscillating hydrofoils with chordwise flexibility.[5,6]

Implementation of the Kutta Condition

In constant potential panel methods for two-dimensional airfoils, for example, Refs. 4 and 7-10, it is common to implement the Kutta condition by setting the difference in potential over the wake to the value

$$\Delta\phi = \phi_N - \phi_1 \qquad (1)$$

where ϕ_N and ϕ_1 are the doublet potential values on the upper and lower panels immediately adjacent to the trailing edge, respectively. The aim is to insure equality of pressure on upper and lower surfaces of the airfoil at the trailing edge; the formulation follows from the requirement for the circulation density to be zero at the trailing edge; see, for example, Ref. 4, p. 243, and Ref. 11.

Using this form of the Kutta condition in a constant potential panel method similar to that described by Moran,[9] but based on the perturbation potential (for details, see Refs. 5 and 6), calculations of the steady flow around airfoil sections showed differences in the pressure coefficient between the upper and lower surfaces at the trailing edge as the angle of attack or the airfoil camber was increased. Table 1 shows how these differences varied with angle of attack and camber. The calculations were done with 40 panels on the section; increasing the number of panels did not reduce these pressure differences. For unsteady large-amplitude motions, there were always large differences in the pressure coefficient at the trailing edge with this Kutta condition.

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An explicit steady flow Kutta condition was implemented by introducing a "dummy" doublet potential ϕ_{N+1} to represent the potential on the upper surface at the trailing edge and equating tangential velocities on the upper and lower panels immediately adjacent to the trailing edge. As first-order differentiation was used for calculation of velocities, this condition was linear in the values of potential and was included as an $N + 1$th equation in the linear system of equations for the potential values on the foil surface (there were N panels on the foil surface arranged in a manner similar to that used by Moran[9]). The $N + 1$th equation was

$$\frac{\phi_{N+1} - \phi_{N-1}}{d_{N-1}} + (u_\infty \cdot t)_N = -\frac{\phi_2 - \phi_1}{d_1} - (u_\infty \cdot t)_1 \qquad (2)$$

where the various ϕ are the values of the potential on the panels; the various d are the distances between control points for the panel indicated by the subscript and the panel with the next higher index value; u_∞ is the uniform inflow velocity; and t is the unit tangential vector at the panel control point. Table 1 shows how this approach reduced the difference in pressure coefficient between upper and lower surfaces on the panels immediately adjacent to the trailing edge to below 10^{-5}. The process affected only the fourth significant figure in the lift, drag, and pitching moment coefficient values. The approach is similar to that used by Yon et al.[11] to overcome errors in calculation of pressure distributions at the trailing edge of thin airfoils, found when using these methods.

In unsteady flow, the panel method calculation proceeded in a series of time steps, and the wake was made up of segments that represented conditions over each time step. The strength of the shed vorticity was determined from the Kutta condition applied at the trailing edge. The wake panels were left in the fluid flow where they were formed. No attempt was made to allow the wake to move with the local induced flow, although for low reduced frequencies this restriction was not expected to lead to large errors. The first wake panel was assumed to leave the trailing edge along the bisector of the trailing-edge angle. The value of the potential on each wake panel was taken to be the mean of the values of $\Delta\phi$ obtained at consecutive time steps on either side of the wake panel under consideration, except that a linear variation of

Table 1 Differences in pressure coefficient between upper and lower surfaces at the trailing edge ΔC_p in panel method calculations on NACA four-digit airfoils with Kutta conditions formed from Eqs. (1) and (2)

Section	Angle of attack, deg	ΔC_p Eq. (1)	ΔC_p Eq. (2)
0012	0.0	2.682×10^{-6}	2.086×10^{-6}
0012	4.0	3.073×10^{-3}	7.033×10^{-6}
0012	8.0	6.065×10^{-3}	2.158×10^{-6}
2412	0.0	-8.257×10^{-3}	-2.384×10^{-6}
2412	4.0	-5.193×10^{-3}	-2.325×10^{-6}
2412	8.0	-2.137×10^{-3}	-9.596×10^{-6}
4412	0.0	-1.624×10^{-2}	-2.980×10^{-7}
4412	4.0	-1.290×10^{-2}	7.391×10^{-6}
4412	8.0	-9.495×10^{-3}	5.603×10^{-6}

potential was applied on the wake panel immediately behind the trailing edge.[12]

Here the steady Kutta condition is insufficient because the pressure is a function of the velocity squared and the rate of change of potential. To account for this, and for crossflow at the trailing edge in both steady and unsteady three-dimensional panel methods, a method based on a Newton-Raphson scheme has been used to iterate to an explicit equal pressure Kutta condition at the trailing edge.[10,12,13] Implementation of this method here showed slow convergence, and a method based on a linearized pressure coefficient was used instead. Again a dummy doublet potential value ϕ_{N+1} was introduced at the upper surface at the trailing edge. The linearized pressure coefficient was included as an $N+1$th equation and solved with the system of linear equations for the potential values. The pressure coefficient in unsteady flow is $C_p = 1 - (q/V)^2 - (2/V^2)\,(\partial\phi/\partial t)$ where q is the local fluid velocity and V is the velocity of the freestream. This was linearized as

$$\left[\frac{\phi_{2p}-\phi_{1p}}{d_1^2}+\frac{2(u_\infty\cdot t)_1}{d_1}\right](\phi_2-\phi_1) + (u_\infty\cdot t)_1^2+\frac{2}{\Delta t}(\phi_1-\phi_{1p}) = \left[\frac{\phi_{(N+1)p}-\phi_{(N-1)p}}{d_{N-1}^2}+\frac{2(u_\infty\cdot t)_N}{d_{N-1}}\right](\phi_{N+1}-\phi_{N-1}) + (u_\infty\cdot t)_N^2+\frac{2}{\Delta t}[\phi_{N+1}-\phi_{(N+1)p}] \tag{3}$$

where the subscript p denotes the value from the previous time step and Δt is the time step. A first-order differentiation was used for the term $\partial\phi/\partial t$.

The actual difference in pressure coefficient between the upper and lower surfaces at the trailing edge was calculated after each step. In most situations this value was small at the first iteration, but if it was larger than a preset value, the calculated values of potential ϕ were used as the "previous" values, and a second solution was obtained. The difference in pressure coefficient at the trailing edge, the number of iterations, and the difference between the potentials ϕ_N and ϕ_{N+1} were monitored throughout the calculations to insure that the differences were always small.

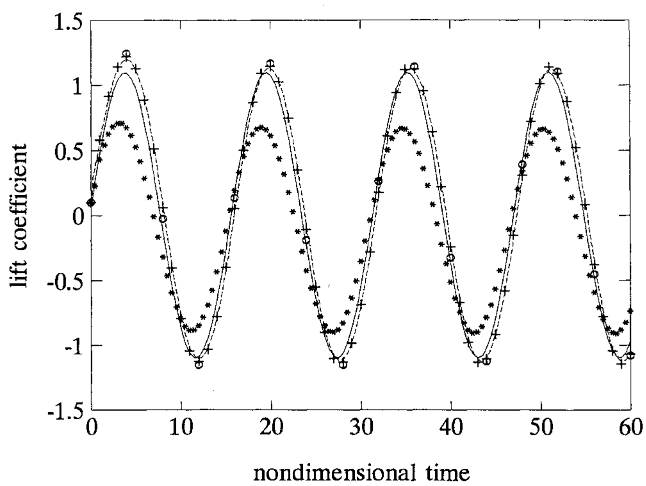

Fig. 1 Lift coefficient for an oscillating airfoil at a reduced frequency of 0.2, with a feathering parameter of 0.4, pitching about its trailing edge with pitch leading heave by $\pi/2$. The solid line is the thin wing small-amplitude solution. The dotted line is the time-domain panel method solution for a NACA 0012 section with a time step of 0.25 s and a Kutta condition in the form of Eq. (3); + and ∘ are for time steps of 1.0 and 4.0 s, respectively. The * show the time-domain solution with a Kutta condition in the form of Eq. (1).

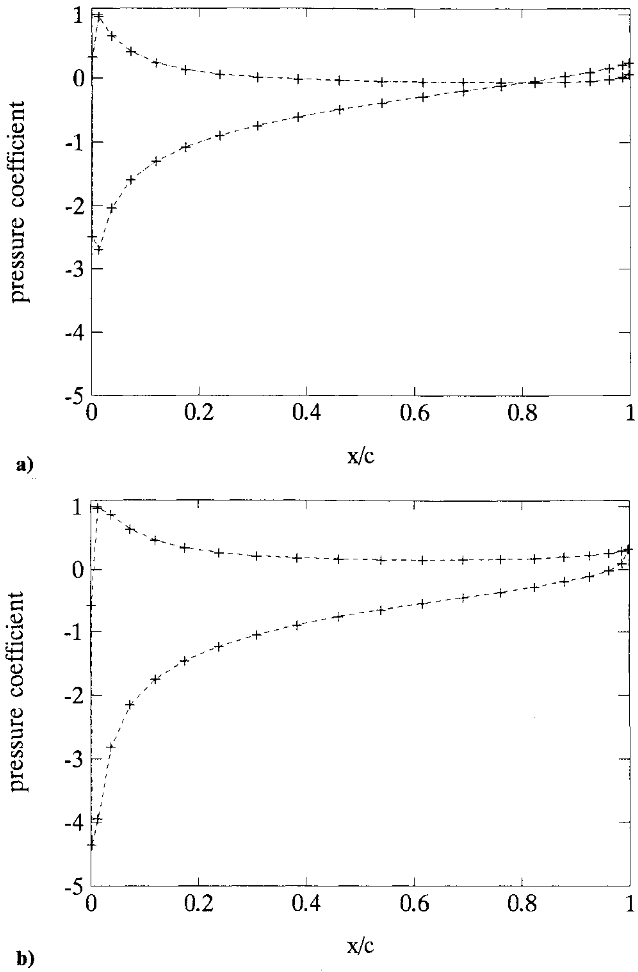

Fig. 2 Pressure distributions for the NACA 0012 section in Fig. 1 at a nondimensional time of 20: a) with a Kutta condition in the form of Eq. (1) and b) with a Kutta condition in the form of Eq. (3).

Calculations were done for the lift coefficient of a NACA 0012 airfoil with 40 panels following a sudden change in angle of attack of 4 deg. The nondimensional time step t_n was 0.1 ($t_n = tV/c$, where t is the time and c is the chord length). Using the Kutta condition represented by Eq. (1), differences in the pressure coefficient between upper and lower panels at the trailing edge were 0.5 at the first time step, reducing at later time steps. Using Eq. (3), the difference in pressure coefficient (actual, not linearized) on the first step was about 0.1 on the first iteration and reduced to less than 0.005 within three iterations; after five steps, the difference in pressure coefficient was less than 0.005 at the first iteration, and it gradually reduced to a value of about 10^{-5} after about $t_n = 9$.

Use of Eq. (1) in calculations for an oscillating airfoil in large-amplitude motion led to large differences in the pressure coefficient at the trailing edge at all time steps. Implementation of the explicit Kutta condition [Eq. (3)] reduced the differences in actual pressure coefficient to below a preset value of 0.005 at all time steps. This occurred either at the first iteration following the implementation of the linearized pressure coefficient or within a few iterations from this condition.

Figure 1 shows the variation in lift coefficient [$C_L = L/(\frac{1}{2}\rho c V^2)$ for a two-dimensional foil, where L is the lift force and ρ is the fluid density] with nondimensional time for a NACA 0012 foil undergoing a sinusoidal oscillation with a heave amplitude ratio $h/c = 1$; a reduced frequency $\sigma = \omega c/(2V)$, of 0.2 (ω is the oscillation frequency); and a feathering parameter $\theta = 0.4$, $\theta = V\alpha/(\omega h)$ where α is the pitch amplitude. The foil had a pitch axis at its trailing edge; pitch led heave by a phase angle of $\pi/2$ rad. Calculations were done with 40 panels. Also shown is the result from thin wing, small-amplitude theory.[14] Apart from a mild transient in the first cycle, the results rapidly converge to a steady-state solution, and with use of Eq. (3), this is within 2.5% of the thin wing solution. The linear variation of potential on the first wake panel made the calculation relatively insensitive to the time step size.[12] With a Kutta condition in the form of Eq. (1), the lift coefficient values were underpredicted by about 30%.

Figure 2 shows the pressure distributions over the section for this motion at the nondimensional time $t_n = 20$: Fig. 2a is with the Kutta condition in the form of Eq. (1); Fig. 2b is with use of Eq. (3). At this time step, the wing is descending with a nosedown pitch and lift is in an upward direction. Use of Eq. (1) leads to a pressure coefficient difference at the trailing edge of about 0.17 and generally lower absolute values of the pressure coefficient (and lower circulation) compared with the calculation done using Eq. (3), which insures equality of pressure on upper and lower surfaces at the trailing edge. Pressure coefficient differences at the trailing edge varied over the range ± 0.3 for this calculation.

Large differences in the pressure coefficient at the trailing edge again became predominant when calculations were done for conditions where the angle of attack of the airfoil was large (above about 15 deg) over a portion of the cycle. However, in real flow, separation would occur in these conditions, so this does not cause a limitation on the method in practice.

Acknowledgment

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Three-Dimensional Closure of the Passage-Averaged Vorticity-Potential Formulation

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Introduction

THE numerical simulation of three-dimensional flows in multistage turbomachines is a difficult subject due to highly rotational viscous effects, further complicated by the unsteady and aperiodic nature of the rotor-stator interaction. The full coupling between the stator and rotor is still not practical within current computational capabilities because of the extraordinary fineness required to capture the flow structure. Thus a reduction of the complexity, based on a simplification of the governing set of equations by appropriate modeling and approximation, is a sensible strategy.

Among such approaches, the through-flow or axisymmetric model has played an important role, and when iteratively coupled with blade-to-blade calculations it leads to a quasi-three-dimensional model.[1–3] The validity of this model is in general dependent on the flow configuration and the accuracy of the blade-to-blade information and empirical data. Numerous attempts for treating these blade-to-blade effects have been carried out[4–6] and such models have been widely used in many turbomachinery design systems.

Recently, fully three-dimensional models for multistage configurations have become an attractive strategy since it is believed that they can provide more accurate predictions than quasi-three-dimensional models. Adamczyk[7] has developed a powerful methodology using sophisticated averaging procedures for each blade row and sets of three-dimensional equations are solved sequentially row by row until a common axisymmetric flow solution is obtained. This approach has shown its advantages and flexibility to simulate flows through multistage turbomachines and to account for the unsteady and aperiodic effects,[8] but requires computer resources not commonly available. Another fast and efficient three-dimensional approach has been proposed by Denton[9] and Dawes[10] for Euler (with a special treatment of viscous effects) and Navier-Stokes equations, respectively. Both assumed that the flow is steady and periodic relative to each of the blade rows individually. The coupling or communication between the blade rows is performed on a mixing plane, which is located between the blade rows where the axisymmetric averaged flow properties are transferred from the upstream to the downstream blade row. This approach limits its ability to handle the variation along the streamwise direction across this mixing plane.

In this Note, a modified three-dimensional model for simulating the rotor-stator interaction flow is described. In this model, fully three-dimensional flows are computed individually in the rotor and stator blade rows. The coupling between blade rows is performed axisymmetrically, but in contrast to Dawes,[10] the axisymmetric properties are obtained using a passage-averaged through-flow calculation instead of averaging the three-dimensional solutions on the mixing plane. Furthermore, the source terms in the through-flow equations are evaluated directly from the three-dimensional solutions for

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